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Modeling dominant height growth of teak plantations in the Caribbean region of Colombia

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ABSTRACT

Identifying sites with adequate biological productivity is a critical factor in ensuring timber production and the profitability of forest-based investments. The productivity of forest sites is influenced by climatic, edaphic and topographic variables, as well as by silvicultural practices. Site index is a phytocentric method widely used to assess site productivity and its estimation is based on dominant height growth modeling. Teak is the fifth most planted forest species in Colombia, and its importance is associated with high economic returns and profitability. This study aims to model dominant height growth using the generalized algebraic difference approach for teak plantations established in the Caribbean region of Colombia. The Lundqvist-Korf model, in which the correlation of the residuals was handled with a continuous autoregressive specification of the first order, resulted in a satisfactory statistical estimation of the dominant height growth. The results indicate that in the Caribbean region of Colombia, productive sites for the establishment of teak plantations can be found as productive as in some tropical American countries and better than some sites in Asian countries. This suggests a potential for the expansion of teak plantations and forest-based investments in Colombia.

Keywords

forest growth, GADA, productivity site index, Tectona grandis

Citation

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INTRODUCTION

The identification and selection of potential sites for commercial reforestation with high biological productivity is a critical factor in forest management. Productive sites ensure adequate timber production, reasonable rotation lengths, and profitability (Schlosser 2023). Forest site productivity refers to the potential of a forest stand to produce timber volume, and it is influenced not only by site factors (e.g., climatic, edaphic, and physiographic variables) but also by silvicultural practices (e.g., site preparation, fertilization, thinning, etc.) (Skovsgaard and Vanclay 2013). Site productivity has been assessed using different approaches, ranging from geocentric to phytocentric methods (Weiskittel et al. 2011). While geocentric methods are based on physical site characteristics, phytocentric methods are based on forest stand characteristics and tree-based metrics (Skovsgaard and Vanclay 2008; Weiskittel et al. 2011).

Site index is a phytocentric method widely used in forestry (Weiskittel et al. 2011), and refers to the mean dominant height of a forest stand at a given base age (Burkhart and Tomé 2012). The site index concept assumes that height growth is positively correlated with volume growth, and that the growth of the highest trees tends to be unaffected by a range of stand densities (Skovsgaard and Vanclay 2008). However, it is important to mention that for some species and site types, these assumptions may not be satisfied (Skovsgaard and Vanclay 2013).

Several approaches are available for estimating the site index using stand variables. The approaches are the Generalized Algebraic Difference Approach (GADA) (Cieszewski and Bailey 2000; Diéguez-Aranda et al. 2006), mixed-effect models (Jerez-Rico et al. 2011; Torres et al. 2012), or the reducible stochastic differential equation (SDE) approach (Garcia 1983; Orrego et al. 2021). In all of these approaches, growth functions are studied by using base mathematical models such as von Bertalanffy, Schumacher, Lundqvist-Korf, and Hossfeld IV, among others.

The GADA method is a generalization of the Algebraic Difference Approach (ADA) proposed by Bailey and Clutter (1974). It consists of the development of dynamic site equations with the properties of base age invariance (i.e., site curves unaffected by choices of base age), and path invariance (i.e., the projection for the mean dominant height from t_0 to t_1 and then from t_1 to t_2 is the same projection from t_0 to t_2) (Cieszewski and Bailey 2000; Diéguez-Aranda et al. 2006). In GADA, the main advantage over the other methods, is that more of one parameter of the base growth function can be considered local or site-specific, and the remaining parameters can be global, allowing for more flexible dynamic equations (Cieszewski 2002). Site quality is represented by local parameters and can, therefore, vary between stands, whereas global parameters are the same for different sites (Diéguez-Aranda et al. 2006). Site-specific parameters are represented as a function of an unobservable and independent variable χ , which describes the site factors that explain height growth (Cieszewski and Bailey 2000), and allows for either polymorphic (i.e., a single asymptote but different growth rates) or anamorphic (i.e., different asymptotes but a common growth rate) curves.

Data from permanent sampling plots and stem analysis have been used to study the growth of the mean dominant height of forest plantations (Tahar et al. 2012; Koirala et al. 2021). Both data sources consist of repeated measurements, either in plots or trees, which inevitably exhibit correlation between observations (Diéguez-Aranda et al. 2006; Tahar et al. 2012). Autocorrelation can affect the parameter estimation of dominant height growth models, leading to potential bias and affecting the variance of the estimated coefficients (Panik 2014). It seems to be reasonable to deal with autocorrelation by considering appropriate correlation structures (e.g. continuous autoregressive specifications) (Tahar et al. 2012), and to ensure reliable predictions of site index values.

Teak (*Tectona grandis*) is a tropical timber in high demand in global markets due to its high durability, dimensional stability, good workability, and high resistance to pests, which allows for multiple uses such as furniture construction, structural elements, flooring, and others (Keogh 2013). This high quality of teak timber is due to its heartwood, which makes it a tropical species of high commercial value (Keogh 2013). The amount of heartwood in teak trees depends on site conditions, management practices, geographic location, tree age, and tree diameter (Moya et al. 2014; Sasidharan and Ramasamy 2021). Therefore, it is important to identify the best sites for planting this species to ensure high-quality teak logs (Moya et al. 2014; Sasidharan and Ramasamy 2021).

India, Thailand and China are the main importers of teakwood as roundwood and sawnwood (Kollert and Walotek 2017; Miassi et al. 2021). Although these countries import teakwood from both forest plantations and natural forests, the majority of imports are from forest plantations. The growing demand for teakwood from forest plantations is driven by conservation policies aimed at reducing logging in natural teak forests in countries such as Myanmar, India, Lao PDR, and Thailand (Kollert and Walotek 2017; Miassi et al. 2021). In addition, India's position as the third largest country in the global construction market has fostered a significant preference for teakwood (ITTO 2024). The dynamics of the global teak market represent an opportunity for tropical countries, where suitable biophysical conditions are favorable for the establishment of commercial teak plantations. As a result, countries such as Ecuador, Costa Rica, Panamá, Ivory Coast, and Ghana are becoming major exporters of teak logs, while Brazil, Tanzania, Indonesia, and Myanmar are major exporters of teak sawnwood (Kollert and Walotek 2017; Miassi et al. 2021).

Colombia could be a major player as a supplier of teakwood. In Colombia, almost 25.9 million hectares of land have been identified for the establishment of forest plantations (Davis et al. 2024). This suitable land is mainly distributed in three regions: Andean (33%), Orinoco (32%) and Caribbean (18%) (Davis et al. 2024). However, to date, only 542,000 hectares are covered by commercial forest plantations, with teak being the fifth most planted species, covering 40,295 hectares mainly in the Caribbean and Andean regions (Ministerio de Agricultura 2023). Teak has been identified by the Colombian government as a promising forest species. It is also listed among the forest species that can receive subsidies from the program known as the Forestry Investment Certificate, which provides economic incentives to activities related to the establishment and management of forest plantations (Ministerio de Agricultura 2022).

Although this species was first planted in the Caribbean region of Colombia more than 80 years ago, only about 19,000 hectares of teak plantations have been established in this region to date (Ministerio de Agricultura 2023). This region is promising for the establishment of new commercial teak plantations due to its proximity to export ports, relatively flat topography with minimal elevation changes, which favors silvicultural practices, and climatic conditions suitable for teak growth (Davis et al. 2024). Furthermore, some studies have shown that teak plantations are an attractive forest-based investment in Colombia, with returns on investment higher than those reported for countries such as Indonesia and Venezuela (Restrepo et al. 2012). However, high profitability requires the selection of suitable sites that guarantee the success in the establishment and management of teak plantations (Jerez-Rico and Andrade 2017).

Mean dominant height growth for teak plantations in Colombia has been modeled using different approaches. Mixed-effects models and an ADA approach were used by Torres et al. (2012). An

SDE approach and a von Bertalanffy base equation were implemented by Orrego et al. (2021), which yielded better results than a GADA approach that did not consider the correlation of repeated measurements. The aim of this study is to model the dominant height growth using the GADA approach for teak plantations established in some sites in the Caribbean region of Colombia. The empirical analysis explicitly considers the correlation structure in the observations as a result of the longitudinal dataset used.

DATA AND METHODS

Study Area

This study was carried out in the teak forest plantations of Tekia S.A.S in the Caribbean region of Colombia, specifically in Puerto Libertador, Department of Córdoba (Figure 1). The study area has an average annual rainfall of 2479 mm with an unimodal pattern, an average annual temperature of 27°C, colluvial-alluvial soils with depths ranging from deep to moderately deep, textures ranging from fine to moderately fine, and well-drained soils (IGAC 2009; Torres et al. 2012).

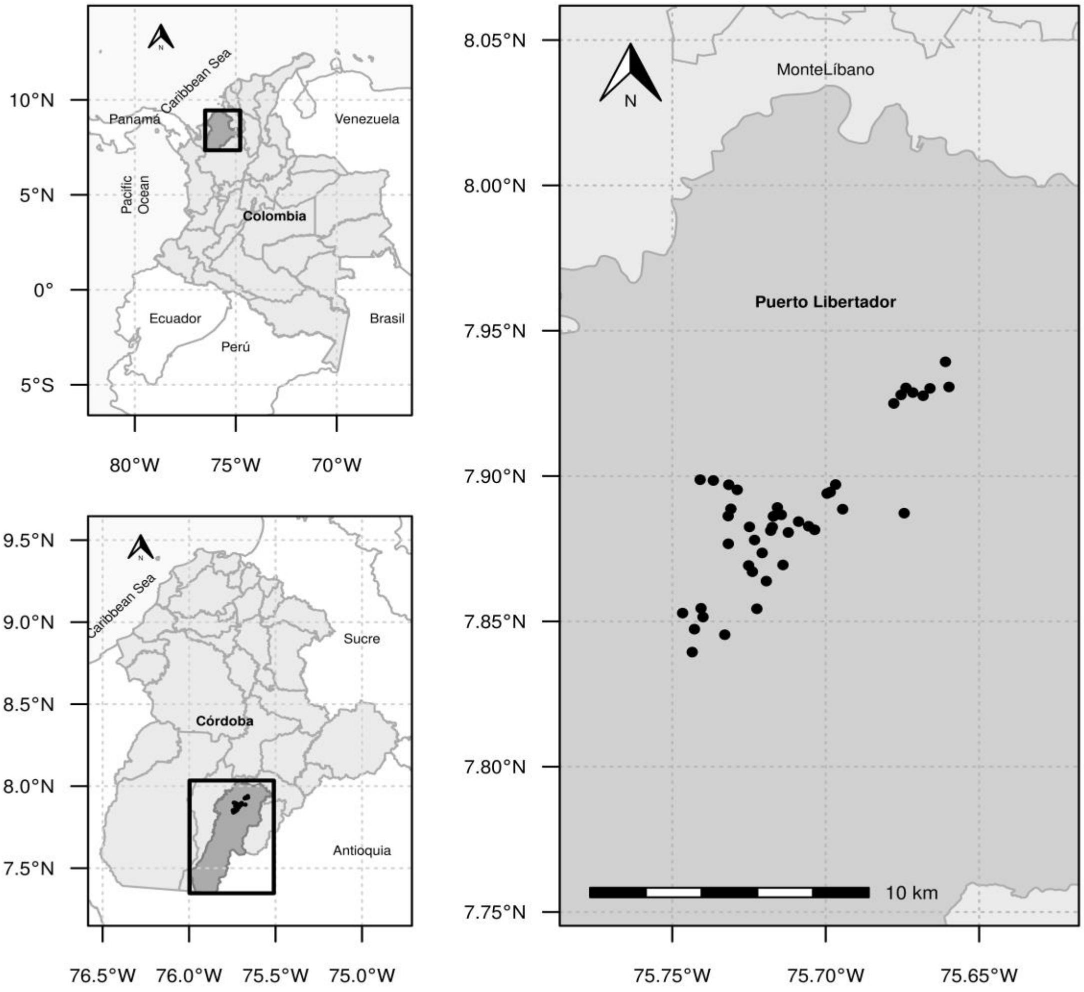

Figure 1. Location of the 44 permanent sampling plots (PSP) established in teak plantations in the Caribbean region of Colombia.

Data collection

Data were collected from 44 permanent sampling plots (PSP) established on these plantations between 1990 and 2011. PSPs were restricted to an elevation range of 71 - 175 m.a.s.l., and had similar silvicultural practices. These practices consisted of a planting density of 1,600 seedlings per hectare, manual weeding for the first two years and annually until the rotation age, pruning at 5 and 9 years, and two thinnings at the ages of 7-9 and 12-13 years, with the goal of maintaining a basal area of approximately 26 m^2 ha^{-1} (Torres et al. 2012). While some PSPs were 600 m^2 in size, others were 1000 m^2.

Height measurements of the tallest trees were collected over 24 years, along with the mean dominant height and age for each measurement, with the number of remeasurements ranging from 3 to 16 per plot, for a total sample size of 440 observations. Mean dominant height was defined as the mean height of the 100 largest trees per hectare, and was used as a proxy variable for site quality, as suggested by several previous studies (Diéguez-Aranda et al. 2006; Torres et al. 2012; Orrego et al. 2021).

Modeling

The mean dominant height (H) growth of teak plantations was modeled using the Lundqvist-Korf and von Bertalanffy models. The Lundqvist-Korf equation, a generalization of the Schumacher model, can be written in its differential form as

$$\frac{1}{H}\frac{dH}{dt} = b\frac{c}{t^{(c+1)}}. \quad (1)$$

It assumes that the relative growth rate of mean dominant height $(1/H)(dH/dt)$ is inversely related to age, and it is influenced by a shape parameter (c), which allows for flexibility in the model (Burkhart and Tomé 2012). The solution of the equation (1) allows for the specification of H as a function of t

$$H = ae^{(b/t^{c})}, \quad (2)$$

where a is the asymptote (m), and denotes the maximum mean dominant height to which a stand tends; b is associated with the growth rate; and c is a shape parameter (Burkhart and Tomé 2012).

The von Bertalanffy equation has a reasonable biological interpretation. Its differential form describes the dominant height growth rate (dH/dt) as a result of the difference between anabolism, ηH^{m}, with $0 < m < 1$, and catabolism, γH (von Bertalanffy 1976; Inga and del Valle 2017)

$$\frac{dH}{dt} = \eta H^{m} - \gamma H. \quad (3)$$

The solution of the equation (3), as suggested by García (2019), can be written as

$$H = a\left[1 - \left\{1 - (H_0/a)^{1/c}\right\}e^{-b(t-t_0)}\right]^c, \quad (4)$$

where a is the asymptote (m); b is the growth intrinsic rate; c is a shape parameter equal to $1/(1-m)$; H_0 and t_0 are initial conditions for mean dominant height and age, respectively. For $H_0 = 0$ and $t = 0$, the equation (4) can be written as

$$H_0 = a(1 - e^{-bt})^c. \quad (5)$$

The Lundqvist-Korf (2) and von Bertalanffy (5) growth equations were used as the base mathematical models for the GADA formulation. This approach was chosen because it allows for dynamic growth curves, with more than one parameter of the base model considered site-specific, which can adequately capture the variability among sites of different quality (Cieszewski and Bailey 2000; Diéguez-Aranda et al. 2006).

For the GADA formulation, the parameters a and b were considered local for the Lundqvist-Korf model, while a and c were considered local for the von Bertalanffy model. The remaining parameters were considered global. Local parameters were defined as a function of the unobservable variable χ (Table 1). The expressions for the local parameters were substituted into the Lundqvist-Korf and von Bertalanffy growth equations, thereby specifying the mean dominant height as a function of age, the global parameters (b_i), and a local parameter (χ_0) ($H = f(t,\, b_i, \chi_0)$) (Table 1). GADA models were estimated using mixed-effects models, assuming χ as a random effect, thus allowing for its variation for each plot.

Table 1. GADA formulation of the Lundqvist-Korf and von Bertalanffy growth equations used to model the dominant height growth of teak plantations in the Caribbean region of Colombia.

Model	**Equation**	**Site-specific parameters**	**GADA formulation**
Lundqvist-Korf	$H = ae^{(b/t^c)}$	$a = e^{\chi}$ $b = b_1\chi$	$H = e^{\chi_0}e^{(b_1\chi_0/t^{b_2})}$
von Bertalanffy	$H = a(1 - e^{-bt})^c$	$a = e^{\chi}$ $c = b_2/\chi$	$H = e^{\chi_0}(1 - e^{-b_1 t})^{b_2/\chi_0}$

Note: H: mean dominant height (m); χ: unobservable variable in the GADA formulations; t: age in years; a: asymptote (m); b: growth intrinsic rate; c: shape parameter. Source: Adapted from (Tahar et al. 2012; Koirala et al. 2021).

Families of site index curves were obtained for the Lundqvist-Korf and von Bertalanffy GADA models (Tahar et al. 2012; Koirala et al. 2021). These curves are generated by solving χ_0 for the initial values of H_0 and t_0, which represent site index and base age, respectively. The solution χ_0, as a function of values H_0 and t_0, was substituted into the corresponding GADA formulation of the models, yielding functions of the type $H = f(t, t_0, H_0)$. For Lundqvist-Korf, this can be expressed as

$$H = H_0 e^{\left(1+b_1 t^{-b_2}\right)/\left(1+b_1 t_0^{-b_2}\right)}, \quad (6)$$

and for von Bertalanffy as

$$H = e^{\chi_0}\left(1 - e^{-b_1 t}\right)^{b_2/\chi_0}, \quad (7)$$

where

$$\chi_0 = \frac{1}{2}\left(\ln H_0 \pm \left((\ln H_0)^2 - 4 b_2 L_0\right)^{1/2}\right),$$

$$L_0 = \ln\left(1 - e^{-b_1 t_0}\right).$$

Because the growth data consisted of successive measurements of mean dominant height and age by plot, it is possible to find serial autocorrelation among observations within each plot (Tahar et al. 2012). Autocorrelation in the residuals of each model was assessed by graphical analysis of the autocorrelation function (ACF), which shows the correlation values of the residuals for a specific given lag order (temporal distance between observations). The existence of autocorrelation was treated by a continuous autoregressive specification of order p (CAR(p)) in the residual terms, as this structure is appropriate for irregularly spaced, unbalanced data (Tahar et al. 2012).

The basic regression assumptions of the models were also tested. Normal distribution of the residuals was assessed by Kolmogorov-Smirnov (K-S) tests, with the null hypothesis of normality. Homoscedasticity was tested by graphical analysis of standardized residuals as a function of the predicted values. Independence of residuals was tested by ACF plots, where correlation values in the first lags should be low and statistically non-significant to ensure no autocorrelation. Models were compared using Akaike's information criterion (AIC), Bayesian information criterion (BIC), precision metrics such as root mean square error (RMSE), mean absolute percentage error (MAPE), average bias (AB), and a performance metric or index of fitting known as model efficiency (ME). A ME value of 100% indicates a "perfect" fit, 0% indicates that the model is no better than the mean, and a negative value indicates a poor model (Burkhart and Tomé 2012). All models and statistical analyses were fitted and run in the *nlme* library of the R software (Pinheiro and Bates 2000; Pinheiro et al. 2021; R Core Team 2024).

RESULTS

Observations of mean dominant height ranged from 5.93 to 27.53 m, and age varied from 1.82 to 24.03 years (Table 2). Significant heterogeneity in mean dominant height growth trajectories for different ages was observed, indicating differences in site quality (Figure 2). In both GADA-estimated models, Lundqvist-Korf and von Bertalanffy, initially fitted without consideration of residual correlation structure, the estimated coefficients were statistically significant ($p < 0.01$) (Table 3). The von Bertalanffy model had lower AIC and BIC values than the Lundqvist-Korf model, suggesting that it was the most appropriate model. Residuals for both models met the assumption of normal distribution based on a K-S test ($p > 0.05$). However, the residuals exhibited autocorrelation for all of the lags, following an exponential decay pattern (Figure 3), and also exhibited heteroscedasticity (Figure 4).

Table 2. Descriptive statistics of the stand variables used to model and validate the dominant heigh growth for teak plantations in the Caribbean region of Colombia.

Variable	Mean	Min	Max	S.D.
Mean dominant height (m)	16.44	5.93	27.53	3.53
Age (years)	9.26	1.82	24.03	4.65

Note: S.D.: Standard deviation.

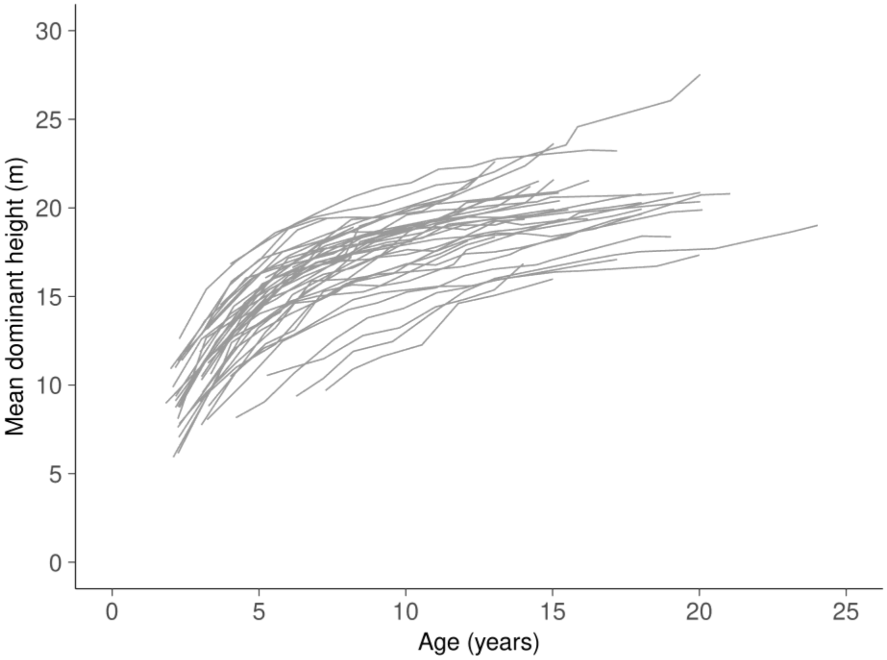

Figure 2. Mean dominant height growth trajectories for teak plantations in the Caribbean region of Colombia.

The CAR(1) specification was satisfactory for dealing with autocorrelation in both models, as the ACF plots showed correlation values close to zero, implying independence of the residuals (Figure 3). The estimated coefficient associated with the CAR(1) specification, ϕ, was 0.98 and 0.97 for the Lundqvist-Korf and von Bertalanffy models, respectively. The estimated coefficients of the GADA models with CAR(1) specification were statistically significant ($p < 0.01$), but their estimated values and corresponding standard errors were different from those obtained without considering the autocorrelation specification (Table 3).

Table 3. Estimated GADA models for the dominant height growth of teak plantations in the Caribbean region of Colombia.

Model	**Parameter**	**S. E.**	**K-S (P-value)**	**AIC**	**BIC**
Lundqvist-Korf	$b_1 = -0.56^{***}$	0.015	0.087	1280	1300
	$b_2 = 0.61^{***}$	0.045			
Lundqvist-Korf CAR(1)	$b_1 = -0.63^{***}$	0.026	0.370	743	768
	$b_2 = 0.73^{***}$	0.041			
	$\phi = 0.98$				
von Bertalanffy	$b_1 = 0.14^{***}$	0.009	0.217	1212	1232
	$b_2 = 2.00^{***}$	0.079			
von Bertalanffy CAR(1)	$b_1 = 0.19^{***}$	0.011	0.189	780	804
	$b_2 = 2.48^{***}$	0.121			
	$\phi = 0.97$				

Note: b_1, b_2: *global parameters for the Lundqvist-Korf and von Bertalanffy GADA models;* ϕ: *first-order parameter of the continuous autoregressive CAR(1) specification applied to residuals; S.E.: standard error of the estimated parameters; ME: model efficiency; K-S: p-value of the Kolmogorov-Smirnov test to assess the normality of the residuals; AIC: Akaike's information criteria; BIC: Bayesian information criterion.* *** *Significance level < 0.01.*

The inclusion of a CAR(1) specification in the GADA models resulted in lower values of AIC and BIC. After accounting for autocorrelation, the Lundqvist-Korf model with CAR(1) outperformed the von Bertalanffy CAR(1) model, which was completely contrary to the results obtained with the estimation without accounting for autocorrelation. Based on the residuals, both models met the normal distribution assumption ($p > 0.05$), and showed homogeneous variance in the predicted values (Figure 4).

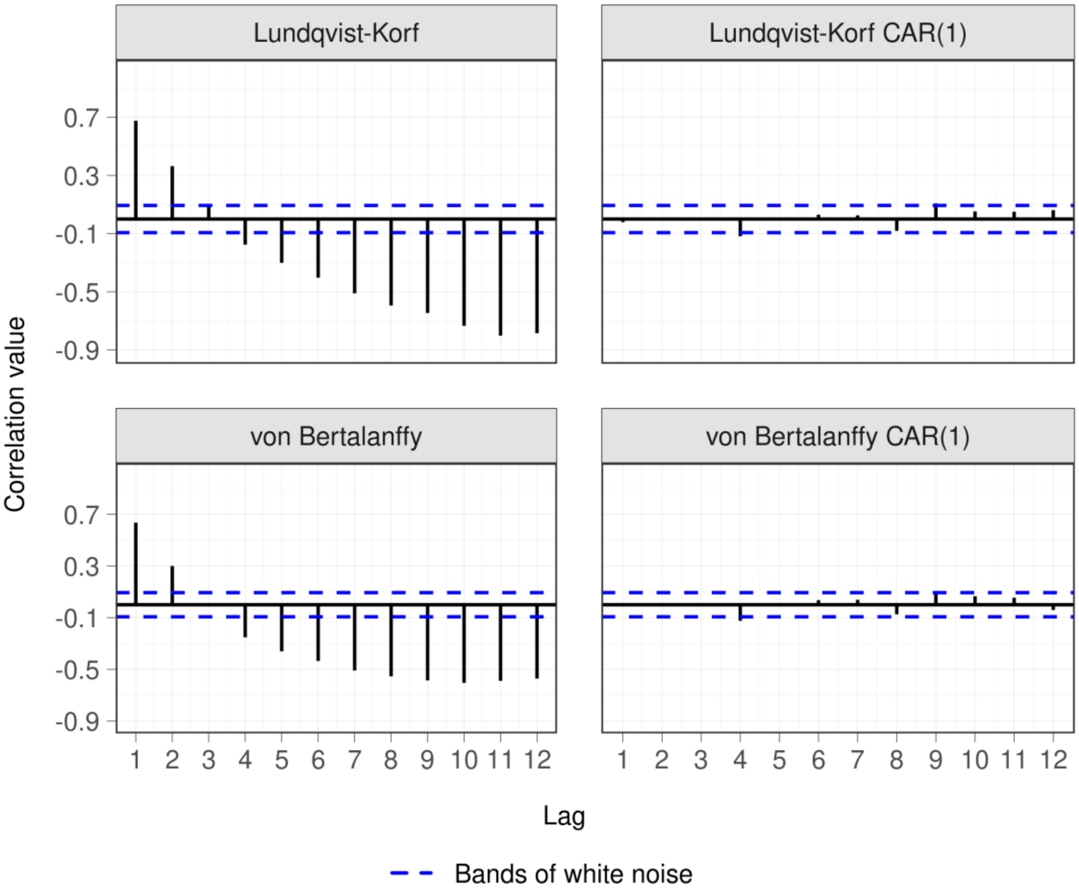

Figure 3. Autocorrelation functions of the residuals of the GADA models without any correlation structure (left panel) and with continuous autoregressive specification CAR(1) (right panel). The white noise bands correspond to the bounds of the correlation values of the independent residuals.

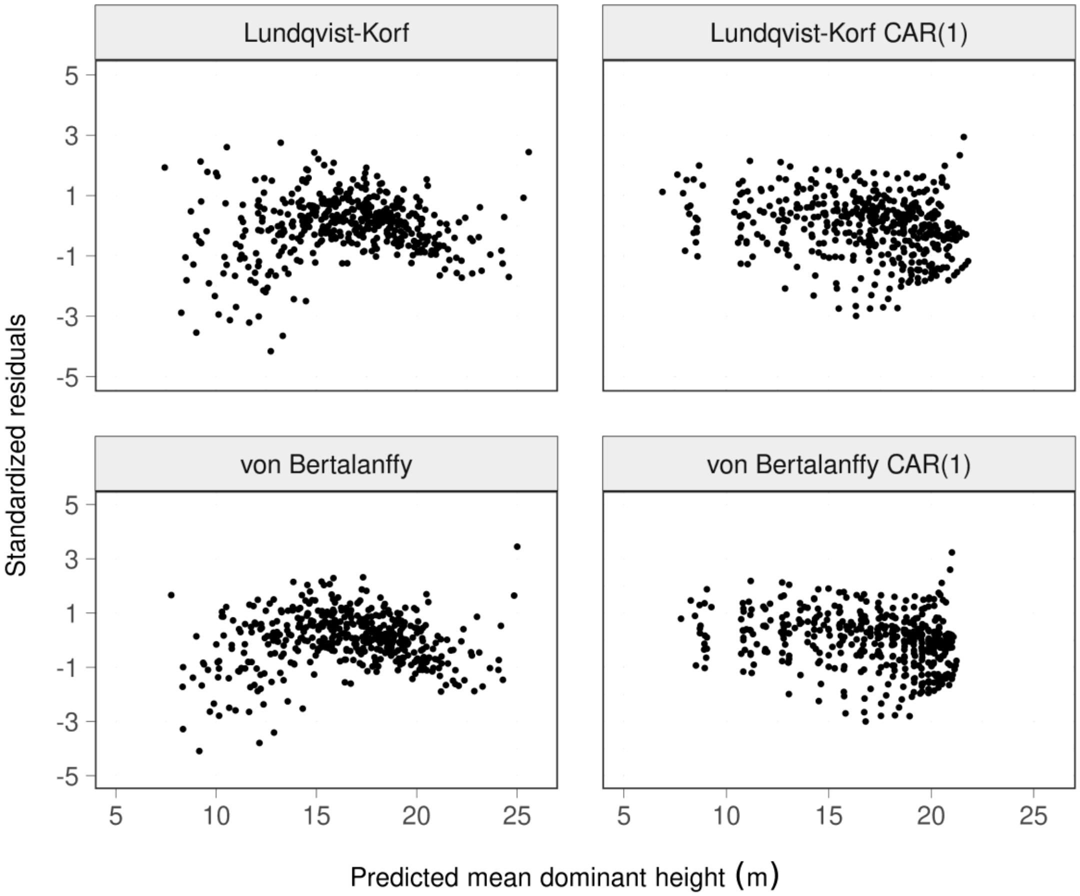

Figure 4. Standardized residuals versus predicted mean dominant height values for the estimated GADA models.

Estimated models with CAR(1) specification resulted in lower values of RMSE, MAPE, and higher ME than the corresponding models without autocorrelation structure. The inclusion of CAR(1) allowed to improve the predictive ability of the models by reducing the prediction errors (Table 4). According to AB, whose values were close to zero, all models showed no bias in the predictions. The Lundqvist-Korf CAR(1) model was selected as the best model for predicting the mean dominant height growth of teak plantations, using the following site index equation

$$H = H_0 e^{(1-0.63t^{-0.73})/(1-0.63t_0^{-0.73})}, \quad (8)$$

where H_0 stands for site index and t_0 for base age.

Mean dominant height growth curves were generated for site indices 13, 16, 19, and 22 m, and for all of the estimated models (Figure 5). In general, the site index curves did not show significant differences. However, some differences were more apparent between the Lundqvist-Korf and von Bertalanffy models.

Table 4. Calculated metrics of precision, bias, and performance calculated for the estimated GADA models of mean dominant height growth of teak plantations in the Caribbean region of Colombia.

Model	RMSE	AB	MAPE (%)	ME (%)
Lundqvist-Korf	0.850	−0.016	4.322	94.81
Lundqvist-Korf CAR(1)	0.467	0.010	2.171	97.55
von Bertalanffy	0.776	−0.009	3.945	95.68
von Bertalanffy CAR(1)	0.495	0.025	2.301	97.24

Note: RMSE: root mean square error; AB: average bias; MAPE: mean absolute percentage error; ME: model efficiency or index of fitting.

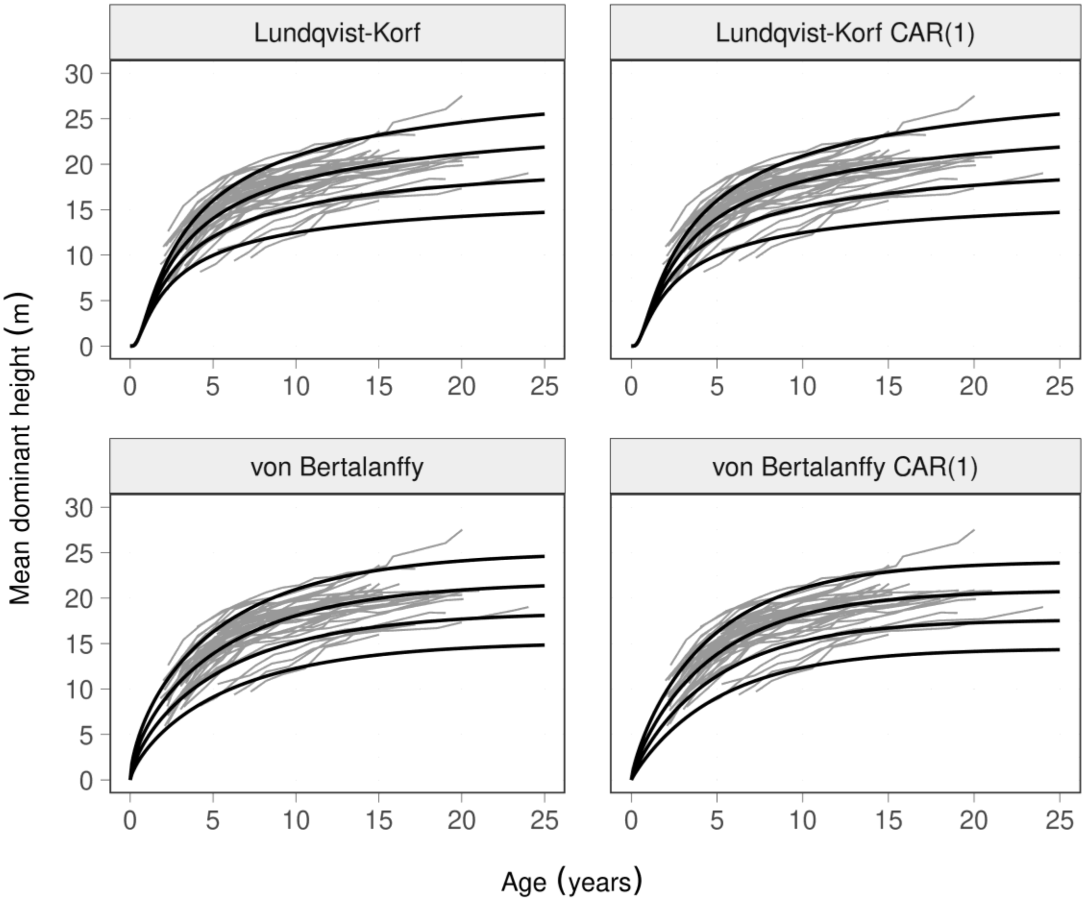

Figure 5. Curve family of predicted mean dominant height growth for site index values of 13, 16, 19, and 22 m at a base age of 12 years, using the estimated parameters of the GADA models.

All estimated models were compared using the Lundqvist-Korf CAR(1) model as the baseline and a site index of 18 m and a base age of 12 years (Figure 6). All estimated models overestimated the predictions of the Lundqvist-Korf CAR(1) model at early ages. After three years, the predictions of the Lundqvist-Korf CAR(1) model were underestimated by the von Bertalanffy models, even after 12 years. Conversely, the Lundqvist-Korf model had similar predictions to its corrected version (CAR(1)) at seven years and longer.

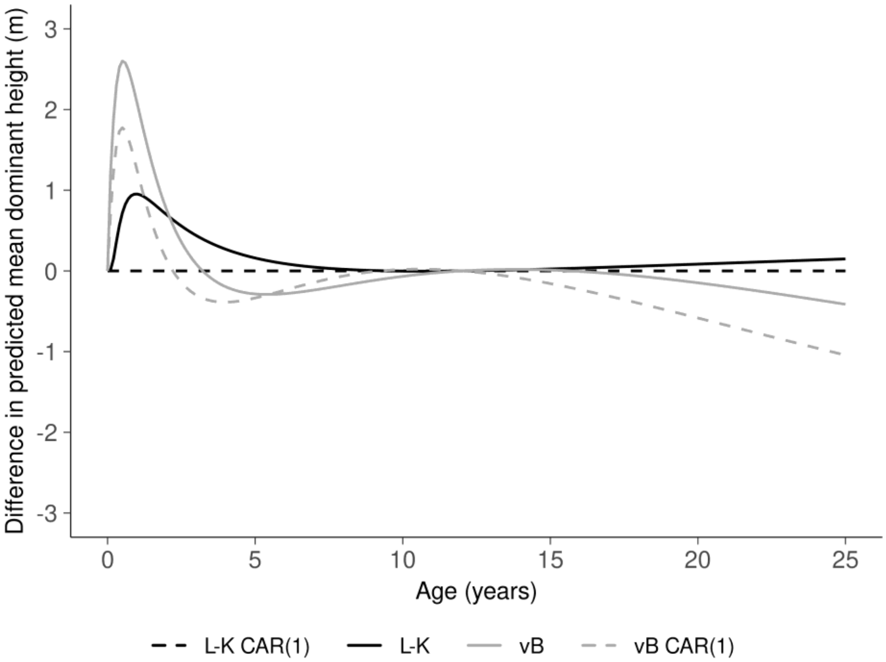

Figure 6. Difference in predicted mean dominant height growth for the Lundqvist-Korf (L-K), von Bertalanffy (vB), and von Bertalanffy CAR(1) models with respect to the Lundqvist-Korf CAR(1) model, for a site index of 18 m and a base age of 12 years.

DISCUSSION

This research found evidence that the Lundqvist-Korf and von Bertalanffy models were suitable to fit the mean dominant height growth of teak plantations located in the Caribbean region of Colombia. The Lundqvist-Korf specification with a residual correlation structure was statistically the best model. The good performance of the Lundqvist-Korf equation may be due to the fact that it is a generalization of a Schumacher equation. This was developed based on the hypothesis that the relative growth rate increases linearly with the inverse square of age (Burkhart and Tomé 2012; Nafidi and El Azri 2021).

Lundqvist-Korf and von Bertalanffy models were also used by Torres et al. (2012) in teak plantations, although they used the ADA formulation and mixed-effects approach as the estimation procedure. They used variance structures to correct for residual heteroscedasticity, and found that the Lundqvist-Korf model performed better than the von Bertalanffy model based on AIC and BIC metrics (Torres et al. 2012). Another previous study fitted the dominant height growth of teak plantations using a von Bertalanffy equation as the base model and different modeling approaches, and found that a stochastic differential equation approach outperformed a GADA specification (Orrego et al. 2021).

Accounting for autocorrelation using autoregressive specifications in GADA models has been widely used to account for the longitudinal structure of data from permanent sampling plots and stem analysis (Diéguez-Aranda et al. 2006; Tewari et al. 2014). It is common that models with correlated residuals tend to underestimate the parameter variance, and to estimate parameters with a slight bias (Glasbey 1980; Panik 2014). This was observed for some parameters of the Hossfeld and King-Prodan type models used to study the dominant height growth of *Pinus nigra* (Seki and Sakici 2017). In our study, after accounting for autocorrelation, the variances of the estimated coefficients were higher compared to the estimated models without accounting for autocorrelation, except for the parameter b_2 in the Lundqvist-Korf model.

We also found that by not accounting for autocorrelation may affect the selection of the best model. The AIC and BIC metrics are affected by the value of the likelihood function, which depends on the estimated coefficient vector and its length (Burnham and Anderson 2002). Including the CAR(1) specification in both models, increased the number of parameters and changed their estimates, giving to the Lundqvist-Korf model the highest predictive ability and the lowest AIC and BIC values.

The high variability of the predicted site index for teak plantations (from 13 to 22 m), indicates that there is considerable heterogeneity associated with the physical factors determining the site productivity. PSPs were not distant from each other, with a mean distance of 4.6 km, and not exceeding 14.4 km. The observed site index variability for nearby PSPs can be explained by soil conditions (e.g., cation exchange capacity, bulk density, etc.) and topographic factors (e.g., elevation, slope), which vary considerably over short distances ($0.1 - 1$ km) (Sasidharan 2021).

The estimated site productivity for teak plantations in the Caribbean region of Colombia was similar to that predicted for some tropical American countries such as Ecuador, Costa Rica, Brazil, Venezuela, and Panama, where the site index can vary from 10 to 26 m for the same base age (12 years) (Table 5). Although the specific management practices were different among these countries, they represent similar geographic conditions. In some Asian countries, such as India and Vietnam, the range of site index values is from 7 to 19 m at the same base age (Table 5). This shows that tropical American countries, including Colombia, may have better sites for establishing teak plantations than Asian countries.

This result is promising and demonstrates the high potential for the reforestation with teak plantations in the Caribbean region of Colombia. In addition, more than 2 million hectares of land have been identified in this region with high suitability for commercial reforestation (UPRA 2014; Davis et al. 2024). Forest-based investments should be focused on sites with high productivity, ensuring high profitability. For the region studied, teak plantations can achieve an average annual increment of 27.8 m^3 ha^{-1} $year^{-1}$ after six years (Restrepo and Orrego 2015). This is higher than the values reported for some Latin American countries (Alvarado and Mata 2013). Finally, teak plantations in the Caribbean region of Colombia may require shorter rotations compared to those reported for teak plantations in Asia, Africa and the Americas (Restrepo and Orrego 2015).

Table 5. Predicted site index values for teak plantations in some tropical Asian and American countries worldwide.

Country	SI_{min}	SI_{max}	Base equation	Source
India	7	19	Sigmoid	Upadhyay et al. 2005
Vietnam	12	18	von Bertalanffy	Huy et al. 2022
Ecuador	10	23	Schumacher	Cañadas et al. 2010
Costa Rica	20	24	Hossfeld	Bermejo et al. 2004
Brazil	16	20	Lundqvist-Korf	Santos et al. 2023
Venezuela	14	24	Schumacher	Jerez-Rico et al. 2011
Panama	16	26	von Bertalanffy	Seppänen and Mäkinen 2020
Colombia	11	23	Lundqvist-Korf	Torres et al. 2020
	13	22	Lundqvist-Korf	This study

SI_{min} and SI_{max} correspond to the minimum and maximum site index, respectively. Both values were projected to a base age of 12 years using the model selected by each study.

CONCLUSIONS

In this study, dynamic site equations were used for teak plantations established in some sites in the Caribbean region of Colombia. The Lundqvist-Korf and von Bertalanffy equations using the GADA formulation were used to model the dominant height growth of teak. These models were successfully fitted, and a CAR(1) specification was considered due to the observed residual correlation, thereby improving the predictive power of the estimated models. The Lundqvist-Korf model with a CAR(1) specification was the best in terms of statistical performance, with the lowest values of AIC, BIC, and RMSE. It is critical to deal with autocorrelation to avoid misleading conclusions about estimated coefficients and model choice. In the Caribbean region of Colombia, there is a high potential for teak plantations due to the existence of sites with site index values ranging from 13 to 22 m, with a base age of 12 years, which is similar to sites in other tropical American countries and better than in Asian countries. This information is useful to identify the best sites for teak plantations and to ensure the profitability of forest-based investments. For future research, GADA models can be improved by including appropriate and spatially explicit topographic and soil variables.

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DATA AVAILABILITY

Data are not publicly available.

CONFLICT OF INTERESTS

The authors declare no conflict of interest.

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